Inter (Part-I) 2021

Mathematics	Group-II	PAPER: I
Time: 2.30 Hours	(SUBJECTIVE TYPE)	Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: (16)

(i) Separate into real and imaginary parts $\frac{2-7i}{4+5i}$.

Ans $\frac{2-7i}{4+5i} = \frac{2-7i}{4+5i} \times \frac{4-5i}{4-5i}$

$$= \frac{(2-7i)(4-5i)}{(4+5i)(4-5i)}$$

$$= \frac{8+35i^2-10i-28i}{(4)^2-(5i)^2}$$

$$= \frac{8+35(-1)-38i}{16-25i^2}$$

$$= \frac{8-35-38i}{16-25(-1)} = \frac{-27-38i}{16+25}$$

$$= \frac{-27-38i}{41} = \frac{-27}{41} - \frac{38}{41}i$$

(ii) Prove that for $\forall z \in c \quad z.\bar{z} = |z|^2$.

Ans Let $z = a + ib$ so that $\bar{z} = a - ib$

$$z.\bar{z} = (a+ib)(a-ib)$$
$$= a^2 - iab + iab - i^2b^2$$
$$= a^2 - (-1)b^2$$
$$= a^2 + b^2 = |z|^2$$

(iii) Find out real and imaginary parts of complex number $(\sqrt{3}+i)^3$.

Ans Let $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$ where

$r^2 = (\sqrt{3})^2 + 1^2$ or $r = \sqrt{3+1} = 2$ and $\theta = \tan^{-1}\frac{1}{\sqrt{3}} = 30^\circ$

So, $(\sqrt{3}+i)^3 = (r\cos\theta + i\,r\sin\theta)^3$

$= r^3(\cos 3\theta + i\sin 3\theta)$ (By De Moivre's Theorem)

$$= 2^3 (\cos 90^\circ + i \sin 90^\circ)$$
$$= 8(0 + i \,.\, 1) = 8i$$

Thus, 0 and 8 are respectively real and imaginary parts of $(\sqrt{3} + i)^3$.

(iv) If G be a group and a, b $\in$ G, then show that $(ab)^{-1} = b^{-1} a^{-1}$.

Ans If a, b are elements of a group G, then show that

$$(ab)^{-1} = b^{-1} a^{-1}$$

Proof:

$$(ab)(b^{-1} a^{-1}) = a (b b^{-1}) a^{-1} \quad \text{(Associative law)}$$
$$= a e a^{-1}$$
$$= a a^{-1}$$
$$= e$$

$\therefore$ ab and $b^{-1} a^{-1}$ are inverse of each other.

(v) Give a table for addition of elements of the set of residue classes modulo 5.

Ans Clearly {0, 1, 2, 3, 4} is the set of residues that we have to consider. We add pairs of elements as in ordinary addition except that when the sum equals or exceeds 5, we divide it out by 5 and insert the remainder only in the table. Thus 4 + 3 = 7 but in place of 7 we insert 2 (= 7 – 5) in the table and in place if 2 + 3 = 5, we insert 0 (= 5 – 5).

$\oplus$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

(vi) Show that $(p \wedge q) \rightarrow p$ is a tautology.

Ans

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

The last column of the above table shows that the statement $(p \wedge q) \rightarrow p$ is true for all values of p and q involved, so $(p \wedge q) \rightarrow p$ is a tautology.

(vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$.

Ans By using the definition of equal matrices, we have

$$x + 3 = y$$
$$x = 3 - y \quad \text{(i)}$$
$$3y - 4 = 2x \quad \text{(ii)}$$

By putting (i) in (ii), we get

$$3y - 4 = 2(3 - y)$$
$$3y - 4 = 6 - 2y$$
$$3y + 2y = 6 + 4$$
$$5y = 10$$
$$y = \frac{10}{5} = 2$$

Now, put y = 2 in (i), we get

$$x = 3 - 2$$
$$x = 1$$

So, {x = 1 and y = 2}

(viii) Find the inverse of $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

Ans Let $A = \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} -2 & 3 \\ -4 & 5 \end{vmatrix} = -2(5) - (-4)(3)$$
$$= -10 + 12$$
$$= 2$$

$$\text{Adj } A = \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$

$$= \frac{1}{2}\begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & \frac{-3}{2} \\ \frac{4}{2} & \frac{-2}{2} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{5}{2} & \frac{-3}{2} \\ 2 & -1 \end{bmatrix}$$

(ix) Without expansion verify that $\begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{vmatrix} = 0.$

Ans L.H.S = $\begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{vmatrix}$

Multiplying all elements of second row by 'abc', we have

$$= \frac{1}{abc}\begin{vmatrix} bc & ca & ab \\ \frac{abc}{a} & \frac{abc}{b} & \frac{abc}{c} \\ a & b & c \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} bc & ca & ab \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

Since all elements of 1^{st} row and 2^{nd} row are identical, so

$$= \frac{1}{abc}(0) = 0 = \text{R.H.S}$$

(x) Convert $x^{1/2} - x^{1/4} - 6 = 0$ into quadratic equation.

Ans This given equation can be written as $(x^{1/4})^2 - x^{1/4} - 6 = 0.$

Let $x^{1/4} = y$

$\therefore$ The given equation becomes

$y^2 - y - 6 = 0$

$\Rightarrow (y - 3)(y + 2) = 0$

$\Rightarrow y = 3,$ or $y = -2$

$\therefore x^{1/4} = 3$	$x^{1/4} = -2$
$\Rightarrow x = (3)^4$	$\Rightarrow x = (-2)^4$
$\Rightarrow x = 81$	$\Rightarrow x = 16$

Hence, solution set is $\{16, 81\}$.

(xi) Evaluate $\left(-1+\sqrt{-3}\right)^5 + \left(-1-\sqrt{-3}\right)^5$.

Ans $\left(-1+\sqrt{-3}\right)^5 + \left(-1-\sqrt{-3}\right)^5$

$$= \left[\frac{2}{2}\left(-1+\sqrt{-3}\right)\right]^5 + \left[\frac{2}{2}\left(-1-\sqrt{3}\right)\right]^5$$

$$= \left[2\left(\frac{-1+\sqrt{-3}}{2}\right)\right]^5 + \left[2\left(\frac{-1-\sqrt{-3}}{2}\right)\right]^5$$

$$= (2\omega)^5 + (2\omega^2)^5$$

$$= 2^5 \cdot \omega^5 + 2^5 \cdot (\omega^2)^5$$
$$= 32\omega^5 + 32\omega^{10}$$
$$= 32(\omega^5 + \omega^{10})$$
$$= 32(\omega^3 \cdot \omega^2 + \omega^6 \cdot \omega^3 \cdot \omega)$$
$$= 32((1) \cdot \omega^2 + (1)(1) \cdot \omega)$$
$$= 32(\omega^2 + \omega)$$
$$= 32(-1) = -32$$

(xii) Discuss the nature of the roots of $2x^2 - 5x + 1 = 0$.

Ans Discriminant $= b^2 - 4ac$

$$= (-5)^2 - 4(2)(1)$$
$$= 25 - 8 = 17$$

$\because$ Disc. of $2x^2 - 5x + 1$ is greater than zero and is not a perfect square. So, its roots are irrational and unequal.

3. Write short answers to any EIGHT (8) questions: (16)

(i) Write $\dfrac{1}{(1-ax)(1-bx)(1-cx)}$ into partial fractions.

Ans Let $\dfrac{1}{(1-ax)(1-bx)(1-cx)} = \dfrac{A}{1-ax} + \dfrac{B}{1-bx} + \dfrac{C}{1-cx}$ (i)

Multiplying both sides of (i) by $(1-ax)(1-bx)(1-cx)$, we get

$1 = A(1-bx)(1-cx) + B(1-ax)(1-cx) + C(1-ax)(1-bx)$ (ii)

Putting $1 - ax = 0$

$(ax = 1 \Rightarrow x = \frac{1}{a})$ in (ii),

$$1 = A\left(1 - b\left(\frac{1}{a}\right)\right)\left(1 - c\left(\frac{1}{a}\right)\right) + B\left(1 - a\left(\frac{1}{a}\right)\right)\left(1 - c\left(\frac{1}{a}\right)\right) + C\left(1 - a\left(\frac{1}{a}\right)\right)\left(1 - b\left(\frac{1}{a}\right)\right)$$

$$1 = A\left(1 - \frac{b}{a}\right)\left(1 - \frac{c}{a}\right) + B(1-1)\left(1 - \frac{c}{a}\right) + C(1-1)\left(1 - \frac{b}{a}\right)$$

$$1 = A\left(1 - \frac{b}{a}\right)\left(1 - \frac{c}{a}\right) + 0 + 0$$

$$1 = A\left(\frac{a-b}{a}\right)\left(\frac{a-c}{a}\right) \Rightarrow \boxed{A = \frac{a^2}{(a-b)(a-c)}}$$

Putting $1 - bx = 0$

$(bx = 1 \Rightarrow x = \frac{1}{b})$ in (ii),

$$1 = A\left(1-b\left(\frac{1}{b}\right)\right)\left(1-c\left(\frac{1}{b}\right)\right)+B\left(1-a\left(\frac{1}{b}\right)\right)\left(1-c\left(\frac{1}{b}\right)\right)$$
$$+C\left(1-a\left(\frac{1}{b}\right)\right)\left(1-b\left(\frac{1}{b}\right)\right)$$

$$1 = A(1-1)\left(1-\frac{c}{b}\right)+B\left(1-\frac{a}{b}\right)\left(1-\frac{c}{b}\right)+C\left(1-\frac{a}{b}\right)(1-1)$$

$$1 = 0+B\left(\frac{b-a}{b}\right)\left(\frac{b-c}{b}\right)+0 \Rightarrow \boxed{B=\frac{b^2}{(b-a)(b-c)}}$$

Putting $1-cx=0$

$$cx = 1 \Rightarrow x=\frac{1}{c} \text{ in (ii)},$$

$$1 = A\left(1-b\left(\frac{1}{c}\right)\right)\left(1-c\left(\frac{1}{c}\right)\right)+B\left(1-a\left(\frac{1}{c}\right)\right)\left(1-c\left(\frac{1}{c}\right)\right)$$
$$+C\left(1-a\left(\frac{1}{c}\right)\right)\left(1-b\left(\frac{1}{c}\right)\right)$$

$$\Rightarrow 1 = A\left(1-\frac{b}{c}\right)(1-1)+B\left(1-\frac{a}{c}\right)(1-1)+C\left(1-\frac{a}{c}\right)\left(1-\frac{b}{c}\right)$$

$$\Rightarrow 1 = 0+0+C\left(\frac{c-a}{c}\right)\left(\frac{c-b}{c}\right)+0 \Rightarrow \boxed{C=\frac{c^2}{(c-a)(c-b)}}$$

By putting these values of A, B, C in (i), we have the required partial fractions.

$$\frac{1}{(1-ax)(1-bx)(1-cx)}$$
$$=\frac{a^2}{(a-b)(a-c)(1-ax)}+\frac{b^2}{(b-a)(b-c)(1-bx)}+\frac{c^2}{(c-a)(c-b)(1-cx)}$$

(ii) Write $\frac{4x^2}{(x^2+1)^2(x-1)}$ into partial fractions.

Ans Let $\frac{4x^2}{(x^2+1)^2(x-1)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{x-1}$

$$\Rightarrow 4x^2 = (Ax+B)(x^2+1)(x-1)+(Cx+D)(x-1)+E(x^2+1)^2 \quad \text{(i)}$$

$$\Rightarrow \quad 4x^2 = (A+E)x^4+(-A+B)x^3+(A-B+C+2E)x^2$$
$$+(-A+B-C+D)x+(-B-D+E) \quad \text{(ii)}$$

Putting $x-1=0 \Rightarrow x=1$ in (i), we get

$$4 = E(1+1)^2 \Rightarrow \boxed{E=1}$$

Equating the coefficients of x^4, x^3, x^2, x, in (ii), we get

$$0 = A + E \quad \Rightarrow \quad A = -E \quad \Rightarrow \quad \boxed{A = -1}$$

$$0 = -A + B \quad \Rightarrow \quad B = A \quad \Rightarrow \quad \boxed{B = -1}$$

$$4 = A - B + C + 2E$$

$$\Rightarrow \quad C = 4 - A + B - 2E = 4 + 1 - 1 - 2 \quad \Rightarrow \quad \boxed{C = 2}$$

$$0 = -A + B - C + D$$

$$\Rightarrow \quad D = A - B + C = -1 + 1 + 2 = 2 \quad \Rightarrow \quad \boxed{D = 2}$$

Hence, partial fractions are: $\dfrac{-x-1}{x^2+1} + \dfrac{2x+2}{(x^2+1)^2} + \dfrac{1}{x-1}$.

(iii) If $a_{n-3} = 2n - 5$, find nth term of the sequence.

Ans Here, $a_{n-3} = 2n - 5$

By putting, n = 4, 5, 6, and 7.

For n = 4,

$$a_{4-3} = 2(4) - 5$$

$$a_1 = 8 - 5$$

$$\boxed{a_1 = 3}$$

For n = 5,

$$a_{5-3} = 2(5) - 5$$

$$a_2 = 10 - 5$$

$$a_2 = 5$$

For n = 6,

$$a_{6-3} = 2(6) - 5 = 12 - 5$$

$$a_3 = 7$$

For n = 7,

$$a_{7-3} = 2(7) - 5 = 14 - 5$$

$$a_4 = 9$$

Thus $a_1 = 3$, $a_2 = 5$, $a_3 = 7$, $a_4 = 9$ is an average progression (A.P).

The common difference

$$d = a_2 - a_1 = a_3 - a_2 = 2$$

$$a_n = a + (n-1)\,d$$

$$= 3 + (n-1)(2) = 3 + 2n - 2$$

$$a_n = 2n + 1$$

(iv) **Find G.M. between $-2i$ and $8i$.**

Ans G.M between $-2i$ and $8i = \pm\sqrt{(-2i)(8i)}$

$$= \pm\sqrt{-16i} = \pm\sqrt{-16(-1)}$$
$$= \pm\sqrt{16}$$
$$= \pm 4$$

So, the G.M between $-2i$ and $8i$ is ± 4.

(v) **If the numbers $\frac{1}{k}, \frac{1}{2k+1}, \frac{1}{4k-1}$ are in H.P., find the value of k.**

Ans $\frac{1}{k}, \frac{1}{2k+1}, \frac{1}{4k-1}$ are in H.P.

$\Rightarrow$ $k, 2k+1, 4k-1$ are in A.P.

$d = (2k+1) - k = (4k-1) - (2k+1)$

$\Rightarrow$ $2k + 1 - k = 4k - 1 - 2k - 1$

$\Rightarrow$ $k + 1 = 2k - 2$

$\Rightarrow$ $2k - 2 - k - 1 = 0$

$\Rightarrow$ $k - 3 = 0$

$\Rightarrow$ $k = 3$

(vi) **Find A, G and H if $a = 2i$, $b = 4i$.**

Ans $A = \frac{a+b}{2}$

$$= \frac{-2i + 8i}{2} = \frac{6i}{2}$$

$A = 3i$

$G = \pm\sqrt{ab}$

$$= \pm\sqrt{(-2i)(8i)} = \pm\sqrt{-16i^2}$$
$$= \pm\sqrt{-16(-1)}$$
$$= \pm\sqrt{16}$$

$G = \pm 4$

$H = \frac{2ab}{a+b}$

$$= \frac{2(-2i)(8i)}{-2i+8i} = \frac{-32i^2}{6i}$$

$H = \frac{-16}{3} i$

(vii) Find the value of n when $^nP_2 = 30$.

Ans

$$^nP_2 = 30$$

$$\frac{n!}{(n-2)!} = 30$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 30$$

$$n(n-1) = 30$$

$$n^2 - n - 30 = 0$$

$$n^2 - 6n + 5n - 30 = 0$$

$$n(n-6) + 5(n-6) = 0$$

$$(n+5)(n-6) = 0$$

$n + 5 = 0$	$n - 6 = 0$
$\Rightarrow n = -5$	$n = 6$

Negative value ignored

So, $^nP_2 = {}^6P_2 = 30$

$$\boxed{n = 6}$$

(viii) Find the number of the diagonals of a 6-sided figure.

Ans A 6-sided figure has 6 vertices. Joining any two vertices, we get a line segment.

$\therefore$ Number of line segments $= {}^6C_2 = \frac{6!}{2!\,4!} = 15$

But these line segments include 6 sides of the figure

$\therefore$ Number of diagonals $= 15 - 6 = 9.$

(ix) A die is rolled. What is the probability that the dots on the top are greater than 4?

Ans $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

The event E that the dots on the top are greater than 4 = $\{5, 6\}$.

$\Rightarrow n(E) = 2$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}.$

(x) Calculate $(9.98)^4$ by using binomial theorem.

Ans $(9.98)^4 = (10 - .02)^4$

$$= 10^4 + \binom{4}{1} 10^3 (-.02) + \binom{4}{2} 10^2 (-.02)^2$$

$$+\binom{4}{3}10(-.02)^3+\binom{4}{4}(-.02)^4$$

$$= 10000 + 4 \times 1000 \times (-.02) + 6 \times 100 \times (.0004)$$

$$+ 4 \times 10 \times (-.000008) + 1 \times (.00000016)$$

$$= 10000 - 80 + 0.24 - 0.00032 + 0.00000016$$

$$= 1000.24000016 - 80.00032 = 9920.23968016$$

(xi) **Expand $(4-3x)^{1/2}$ up to 4 terms by using binomial theorem.**

Ans $(4-3x)^{1/2} = 4^{1/2}\left(1-\frac{3x}{4}\right)^{1/2}$

$$= 2\left(1-\frac{3x}{4}\right)^{½}$$

$$= 2\left\{1+\left(\frac{1}{2}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{-3x}{4}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{-3x}{4}\right)^3+\cdots\right\}$$

$$= 2-\frac{3}{4}x-\frac{9}{64}x^2-\frac{27}{512}x^3 ------$$

valid if $|x| < \frac{4}{3}$

(xii) **Evaluate $^{12}C_3$.**

Ans $$^nC_r = \frac{n!}{(n-r)!\,r!} \quad \text{(i)}$$

By putting n = 12, r = 3 in (i), we have

$$^{12}C_3 = \frac{12!}{(12-3)!\,3!}$$

$$= \frac{12!}{9!\,3!}$$

$$= \frac{12 \times 11 \times 10 \times 9!}{9!\ 3 \times 2 \times 1} = 4 \times 11 \times 5$$

$$= 220$$

4. **Write short answers to any NINE (9) questions: (18)**

(i) **Convert 75° 6′ 30° into radians.**

Ans $75^\circ\, 6'\, 30'' = 75^\circ\left(6+\frac{30}{60}\right)'$

$$= \left(75+\frac{13}{2 \times 60}\right)^\circ$$

$$= \left(\frac{9013}{120}\right)^{o}$$

$$= \frac{9013}{120} \times \frac{\pi}{180} \text{ radians}$$

$$= \frac{9013}{21600} \text{ radians}$$

$$\simeq 75.1083\,(0.01745)$$

$$\simeq 1.3106 \text{ radians.}$$

(ii) Evaluate $\dfrac{1-\tan^2\left(\frac{\pi}{3}\right)}{1+\tan^2\left(\frac{\pi}{3}\right)}$.

Ans $$\frac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}} = \frac{1-(\sqrt{3})^2}{1+(\sqrt{3})^2}$$

$$= \frac{1-3}{1+3} = \frac{-2}{4} = -\frac{1}{2}$$

Hence $$\frac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}} = -\frac{1}{2}$$

(iii) Prove that $\sec^2 A + \operatorname{cosec}^2 (A) = \sec^2 (A) \operatorname{cosec}^2 (A)$ where $\left(A \neq \frac{n\pi}{2}, n \in z\right)$.

Ans L.H.S $= \sec^2 A + \operatorname{cosec}^2 A$

$$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A}$$

$$= \frac{1}{\cos^2 A \sin^2 A} \qquad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{1}{\cos^2 A} \cdot \frac{1}{\sin^2 A}$$

$$= \sec^2 A \,.\, \operatorname{cosec}^2 A = \text{R.H.S}$$

Hence $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \,.\, \operatorname{cosec}^2 A$

(iv) Prove that $\tan (180^o + \theta) = \tan \theta$

Ans L.H.S $= \tan (180^o + \theta)$

$$= \frac{\tan 180^\circ + \tan\theta}{1 - \tan 180^\circ \tan\theta}$$

$$= \frac{0 + \tan\theta}{1 - 0\,(\tan\theta)} = \tan\theta = \text{R.H.S}$$

(v) Prove that $\cot(\alpha + \beta) = \dfrac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$.

Ans L.H.S $= \cot(\alpha + \beta) = \dfrac{1}{\tan(\alpha + \beta)}$

$$= \frac{1}{\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$$

$$= \frac{1 - \tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$$

$$= \frac{1 - \dfrac{1}{\cot\alpha}\dfrac{1}{\cot\beta}}{\dfrac{1}{\cot\alpha} + \dfrac{1}{\cot\beta}}$$

$$= \frac{\dfrac{\cot\alpha\cot\beta - 1}{\cot\alpha\cot\beta}}{\dfrac{\cot\beta + \cot\alpha}{\cot\alpha\cot\beta}}$$

$$= \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$$

= R.H.S Proved.

(vi) Prove that $\dfrac{\sin 2\alpha}{1 + \cos 2\alpha} = \tan\alpha$.

Ans L.H.S $= \dfrac{\sin 2\alpha}{1 + \cos 2\alpha}$

$$= \frac{2\sin\alpha\cos\alpha}{1 + 2\cos^2\alpha - 1} = \frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha}$$

$$= \frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \text{R.H.S}.$$

(vii) Find the period of $\tan\dfrac{x}{7}$.

Ans $\tan\dfrac{x}{7} = \tan\dfrac{1}{7}[x + 7(\pi)]$

$= \tan (x + \pi)$

$= \tan x$

So, the period of $\tan \frac{x}{7}$ is 7π.

(viii) In ΔABC if $\beta = 60^\circ$, $\gamma = 15^\circ$ and $b = \sqrt{6}$, then find 'c'.

Ans $\beta = 60^\circ, \gamma = 15^\circ, b = \sqrt{6}$

$c = ?$

As $\alpha + \beta + \gamma = 180^\circ \Rightarrow \alpha + 60^\circ + 15^\circ = 180^\circ$

$\alpha = 180^\circ - 75^\circ = 105^\circ$

As $\frac{c}{\sin \gamma} = \frac{b}{\sin \beta}$ (law of sines)

$$c = \frac{b}{\sin \beta} \cdot \sin \gamma$$

$$= \frac{\sqrt{6}}{\sin 60^\circ} \cdot \sin 15^\circ$$

$$c = \frac{\sqrt{6}}{.866} \times 0.25882$$

$$c = 0.7320$$

(ix) In ΔABC if $a = 34$, $b = 20$ and $c = 42$, find angle 'r'.

Ans

$$s = \frac{a + b + c}{2}$$

$$= \frac{34 + 20 + 42}{2} = \frac{96}{2} = 48$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{48(48-34)(48-20)(48-42)}$$

$$\Delta = 336$$

$$r = \frac{\Delta}{s}$$

$$r = \frac{336}{48} = 7$$

(x) Show that $r = (s - a) \tan \left(\frac{\alpha}{2}\right)$.

Ans To prove $r = (s - a) \tan \frac{\alpha}{2}$

We know that: $\tan \frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

$$\text{R.H.S} = (s-a)\tan\frac{\alpha}{2} = (s-a)\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$= \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$$= \sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2}} = \frac{\Delta}{s} = r$$

$$\therefore \quad (s-a)\tan\frac{\alpha}{2} = r$$

(xi) Show that $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$.

Ans $\pi = \cos^{-1}(-1) = \cos^{-1}(-x^2 - 1 + x^2)$

$$= \cos^{-1}(-x^2 - (1 - x^2))$$

$$= \cos^{-1}\left((-x)(x) - \sqrt{(1-x^2)(1-x^2)}\right)$$

$$= \cos^{-1}(-x) + \cos^{-1} x$$

$$\because \cos^{-1} A + \cos^{-1} B = \left(AB - \sqrt{(1-A^2)(1-B^2)}\right)$$

$$= \cos^{-1}(-x) = \pi - \cos^{-1} x$$

Hence proved.

(xii) Find the value of $\sec\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$.

Ans We first find the value of y, whose sine is $-\frac{1}{2}$.

$$\sin y = -\frac{1}{2}, \qquad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

$$\Rightarrow \quad y = -\frac{\pi}{6}$$

$$\Rightarrow \quad \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\therefore \quad \sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \frac{2}{\sqrt{3}}$$

(xiii) Find the solution of $\operatorname{cosec}\theta = 2$ which lie in $[0, 2\pi]$.

Ans $\operatorname{cosec}\theta = 2$

or $\quad \frac{1}{\operatorname{cosec}\theta} = \frac{1}{2}$

$$\Rightarrow \quad \sin\theta = \frac{1}{2}$$

$\therefore$ $\sin\theta$ is positive in first and second quadrants with the angle $\theta = \frac{\pi}{6}$.

$$\therefore \quad \theta = \frac{\pi}{6}$$

$$\text{and} \quad \theta = \pi - \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6}$$

$$\frac{\pi}{6}, \frac{5\pi}{6}$$

SECTION-II

NOTE: Attempt any Three (3) questions.

Q.5.(a) Solve the system of equations by Cramer's rule
$2x + 2y + z = 3$, $3x - 2y - 2z = 1$, $5x + y - 3z = 2$. (5)

Ans Given system of equations:

$$2x + 2y + z = 3$$
$$3x - 2y - 2z = 1$$
$$5x + y - 3z = 2$$

$$\begin{bmatrix} 2 & 2 & 1 \\ 3 & -2 & -2 \\ 5 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$AX = B$$

$$|A| = \begin{vmatrix} 2 & 2 & 1 \\ 3 & -2 & -2 \\ 5 & 1 & -3 \end{vmatrix}$$

$$= 2\begin{vmatrix} -2 & -2 \\ 1 & -3 \end{vmatrix} - 2\begin{vmatrix} 3 & -2 \\ 5 & -3 \end{vmatrix} + 1\begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix}$$

$$= 2(6 + 2) - 2(-9 + 10) + 1(3 + 10)$$

$$= 2(8) - 2(1) + 1(13) = 16 - 2 + 13 = 27$$

By Cramer's Rule:

$$|A_x| = \begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= 3\begin{vmatrix} -2 & -2 \\ 1 & -3 \end{vmatrix} - 2\begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}$$

$$= 3(6 + 2) - 2(-3 + 4) + 1(1 + 4)$$

$$= 3(8) - 2(1) + 1(5) = 24 - 2 + 5 = 27$$

$$|A_y| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} - 3\begin{vmatrix} 3 & -2 \\ 5 & -3 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix}$$

$$= 2(-3 + 4) - 3(-9 + 10) + 1(6 - 5)$$

$$= 2(1) - 3(1) + 1(1) = 2 - 3 + 1 = 0$$

$$|A_z| = \begin{vmatrix} 2 & 2 & 3 \\ 3 & -2 & 1 \\ 5 & 1 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} + 3\begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix}$$

$$= 2(-4 - 1) - 2(6 - 5) + 3(3 + 10)$$

$$= 2(-5) - 2(1) + 3(13) = -10 - 2 + 39 = 27$$

$$x = \frac{|A_x|}{|A|} = \frac{27}{27} = 1$$

$$y = \frac{|A_y|}{|A|} = \frac{0}{27} = 0$$

$$z = \frac{|A_z|}{|A|} = \frac{27}{27} = 1$$

So, the solution set is:

$$\{x = 1, y = 0, z = 1\}$$

(b) Solve the system of equations

$2x - y = 4$; $2x^2 - 4xy - y^2 = 6$ **(5)**

Ans Let $y = 2x - 4$ ... (i)

$2x^2 - 4xy - y^2 = 6$ ... (ii)

Putting $y = 2(x - 2)$ in (ii), we get

$$2x^2 - 4x \times (2x - 4) - (2x - 4)^2 = 6$$

$$2x^2 - 8x^2 + 16x - (4x^2 - 16x + 16) - 6 = 0$$

$$-6x^2 + 16x - 4x^2 + 16x - 16 - 6 = 0$$

$$-10x^2 + 32x - 22 = 0$$

$5x^2 - 16x + 11 = 0$ gives

$$x = \frac{16 + 6}{10} \quad \text{or} \quad x = \frac{16 - 6}{10}$$

$$\Rightarrow \quad x = \frac{11}{5} \quad \text{or} \quad x = 1$$

$$x = \frac{16+6}{10} \quad \text{or} \quad x = \frac{16-6}{10}$$

$$\Rightarrow \quad x = \frac{11}{5} \quad \text{or} \quad x = 1$$

If $x = \frac{11}{5}$, then $y = 2\left(\frac{11}{5}\right) - 4 = \frac{22}{5} - 4 = \frac{2}{5}$

If $x = 1$, then $y = 2(1) - 4 = -2$

Hence, $S.S = \left\{\left(\frac{11}{5}, \frac{2}{5}\right), (1, -2)\right\}$.

Q.6.(a) Resolve $\frac{x-1}{(x-2)(x+1)^3}$ into partial fraction. (5)

Ans Let $\frac{x-1}{(x-2)(x+1)^3} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$ (i)

Multiplying both sides with $(x-2)(x+1)^3$, we get

$x - 1 = A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1) + D(x-2)$ (ii)

Putting $x = 2$ in (ii), we get

$2 - 1 = A(2+1)^3 + B(2-2)(2+1)^2 + C(2-2)(2+1) + D(2-2)$

$$1 = 27A + 0 + 0 + 0 \quad \Rightarrow \quad \boxed{A = \frac{1}{27}}$$

Putting $x = -1$ in (ii), we get

$-1 - 1 = A(-1+1)^3 + B(-1-2)(-1+1)^2 + C(-1-2)(-1+1) + D(-1-2)$

$$-2 = 0 + 0 + 0 - 3D \quad \Rightarrow \quad \boxed{D = \frac{2}{3}}$$

Now (ii) can be written as

$x - 1 = A(x^3 + 3x^2 + 3x + 1) + B(x-2)(x^2 + 2x + 1) + C(x^2 - x - 2) + D(x - 2)$

$x - 1 = A(x^3 + 3x^2 + 3x + 1) + B(x^3 + 2x^2 + x - 2x^2 - 4x - 2) + C(x^2 - x - 2) + D(x - 2)$

$= A(x^3 + 3x^2 + 3x + 1) + B(x^3 - 3x - 2) + C(x^2 - x - 2) + D(x - 2)$

$= (Ax^3 + 3Ax^2 + 3Ax + A) + (Bx^3 - 3Bx - 2B) + (Cx^2 - Cx - 2C) + Dx - 2D$

$x - 1 = (A + B)x^3 + (3A + C)x^2 + (3A - 3B - C + D)x + A - 2B - 2C - 2D$

By comparing coefficients of x^3, x^2, x, x^0 on both sides, we have

$A + B = 0 \quad$ (iii) $\qquad 3A - 3B - C + D = 1 \quad$ (iv)

$3A + C = 0 \quad$ (v) $\qquad A - 2B - 2C - 2D = -1 \quad$ (vi)

Putting $A = \frac{1}{27}$ in (iii), we have

$$\frac{1}{27} + B = 0 \quad \Rightarrow \quad \boxed{B = -\frac{1}{27}}$$

Putting $A = \frac{1}{27}$ in (iv), we have

$$3\left(\frac{1}{27}\right) + C = 0 \quad \Rightarrow \quad \boxed{C = -\frac{1}{9}}$$

Putting these values of A, B, C, D in (i), we have

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{\frac{1}{27}}{x-2} + \frac{-\frac{1}{27}}{x+1} + \frac{-\frac{1}{9}}{(x+1)^2} + \frac{\frac{2}{3}}{(x+1)^3}$$

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{1}{27(x-2)} - \frac{1}{27(x+1)} - \frac{1}{9(x+1)^2} + \frac{2}{3(x+1)^3}$$

(b) Find four A.Ms between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$.

Ans Let A_1, A_2, A_3, A_4 be four A.Ms. between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$.

So, $\sqrt{2}, A_1, A_2, A_3, A_4, \frac{12}{\sqrt{2}}$ are in A.P.

Given $a_1 = \sqrt{2}$ and $a_6 = \frac{12}{\sqrt{2}}$,

So using $a_n = a_1 + (n-1)d$, we have

$$\frac{12}{\sqrt{2}} = \sqrt{2} + (6-1)d \qquad (\because n = 6)$$

$$\frac{12}{\sqrt{2}} - \sqrt{2} = 5d \text{ gives} \qquad \frac{10}{\sqrt{2}} = 5d$$

$$\Rightarrow \qquad d = \frac{10}{5\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Now $\quad A_1 = a_1 + d = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

$A_2 = a_1 + 2d = \sqrt{2} + 2(\sqrt{2}) = 3\sqrt{2}$

$A_3 = a_1 + 3d = \sqrt{2} + 3(\sqrt{2}) = 4\sqrt{2},$

$$A_4 = a_1 + 4d = \sqrt{2} + 4(\sqrt{2}) = 5\sqrt{2}$$

Hence, the four A.Ms. between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$ are $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$, $5\sqrt{2}$

Q.7.(a) Find the values of n and r when $^nC_r = 35$ and $^nP_r = 210$. **(5)**

Ans For Answer See Paper 2017 (Group-I), Q.3(viii).

(b) Find the term involving x^4 in the expansion of $(3 - 2x)^7$. **(5)**

Ans For Answer See Paper 2017 (Group-I), Q.7.(b).

Q.8.(a) Prove that $\frac{1 + \cos\theta}{1 - \cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$. **(5)**

Ans R.H.S $= (\operatorname{cosec}\theta + \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1 + \cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1 + \cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1 + \cos\theta)^2}{1 - \sin^2\theta}$$

$$= \frac{(1 + \cos\theta)(1 + \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$$

$$= \frac{1 + \cos\theta}{1 - \cos\theta} = \text{L.H.S}$$

(b) Prove that $\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$. **(5)**

Ans $\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\cos\theta} = \frac{\cos 3\theta \sin\theta + \sin 3\theta \cos\theta}{\sin\theta\cos\theta}$

$$= \frac{\sin(3\theta + \theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin 4\theta}{\sin\theta\cos\theta}$$

$$= \frac{2\sin 2\theta \cos 2\theta}{\sin\theta\cos\theta}$$
$$= \frac{4\sin\theta\cos\theta\cos 2\theta}{\sin\theta\cos\theta}$$
$$= 4\cos 2\theta$$
$$= \text{R.H.S}$$

Q.9.(a) Prove that $(r_1 + r_2)\tan\left(\frac{\gamma}{2}\right) = c.$ **(5)**

Ans L.H.S $= (r_1 + r_2)\tan\frac{\gamma}{2}$

$$= \frac{\Delta}{s-a} + \frac{\Delta}{s-b}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$
$$= \left(\frac{\Delta(s-b) + \Delta(s-a)}{(s-a)(s-b)}\right)\sqrt{\frac{s(s-c)(s-a)(s-b)}{s^2(s-c)^2}}$$
$$= \Delta\left(\frac{s-b+s-a}{(s-a)(s-b)}\right)\frac{\sqrt{s(s-c)(s-a)(s-b)}}{s(s-c)}$$
$$= \Delta\left(\frac{2s-a-b-c+c}{(s-a)(s-b)}\right)\frac{\Delta}{s(s-c)}$$
$$= \Delta^2\left(\frac{2s-(a+b+c)+c}{s(s-a)(s-b)(s-c)}\right)$$
$$= \Delta^2\left(\frac{2s-(2s)+c}{\Delta^2}\right) = c = \text{R.H.S}$$

(b) Prove that $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}.$ **(5)**

Ans Let $\alpha = \sin^{-1}\frac{5}{13}$

$\Rightarrow \quad \sin\alpha = \frac{5}{13}$

And $\quad \beta = \sin^{-1}\frac{7}{25}$

$\Rightarrow \quad \sin\beta = \frac{7}{25}$

$\because$ Both $\sin\alpha$ and $\sin\beta$ are positive,

$\therefore \quad \alpha, \beta \in \left[0, \frac{\pi}{2}\right]$

$\sin^2\alpha + \cos^2\alpha = 1$

$$\cos^2\alpha = 1 - \sin^2\alpha$$

$$\sqrt{\cos^2\alpha} = \sqrt{1 - \sin^2\alpha}$$

$$\cos\alpha = \sqrt{1 - \left(\frac{5}{13}\right)^2}$$

$$= 1 - \frac{25}{169} = \sqrt{\frac{144}{169}} = \frac{12}{13}$$

Similarly, $\cos\beta = \sqrt{1 - \sin^2\beta}$

$$= \sqrt{1 - \left(\frac{7}{25}\right)^2}$$

$$= \sqrt{1 - \frac{49}{625}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$$

Finally,

$$\cos\left(\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25}\right) = \cos(\alpha + \beta)$$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \left(\frac{12}{13}\right)\left(\frac{24}{25}\right) - \left(\frac{5}{13}\right)\left(\frac{7}{25}\right)$$

$$= \frac{288}{325} - \frac{35}{325} = \frac{288 - 35}{325}$$

$$\cos\left(\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25}\right) = \frac{253}{325}$$

$$\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$$ Proved.